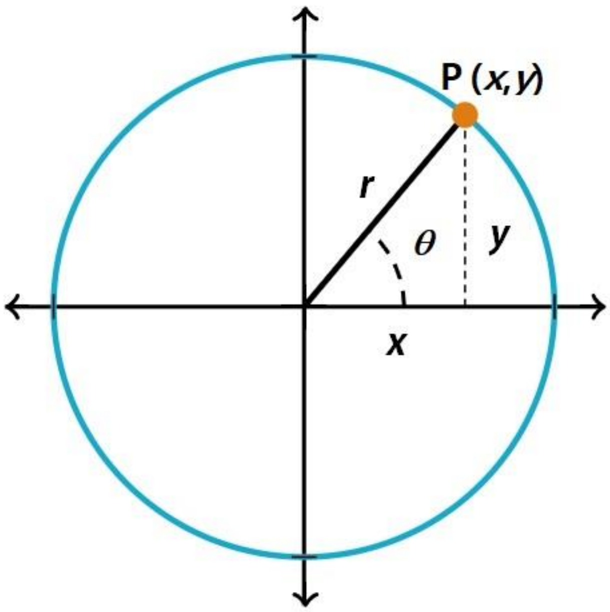
Trigonometry Sheet

1 Definitions



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\text{cosec}\,\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

2 Reciprocals

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

3 Radians

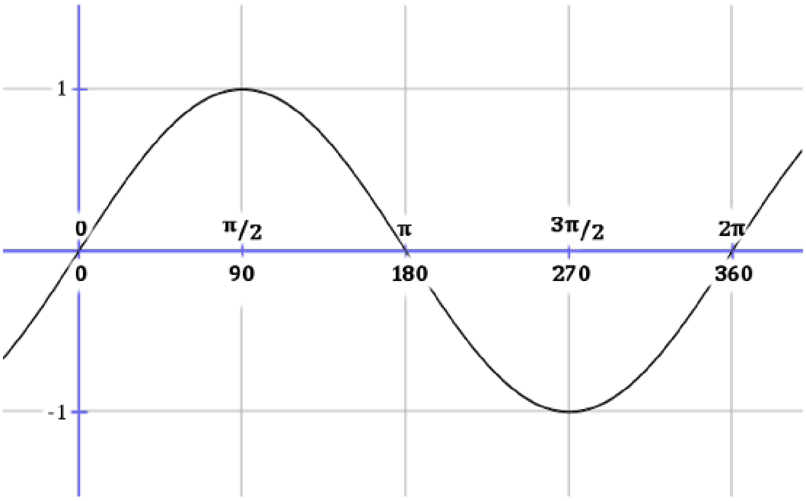
$30° = \frac{\pi}{6}$ $45° = \frac{\pi}{4}$

$60° = \frac{\pi}{3}$ $90° = \frac{\pi}{2}$

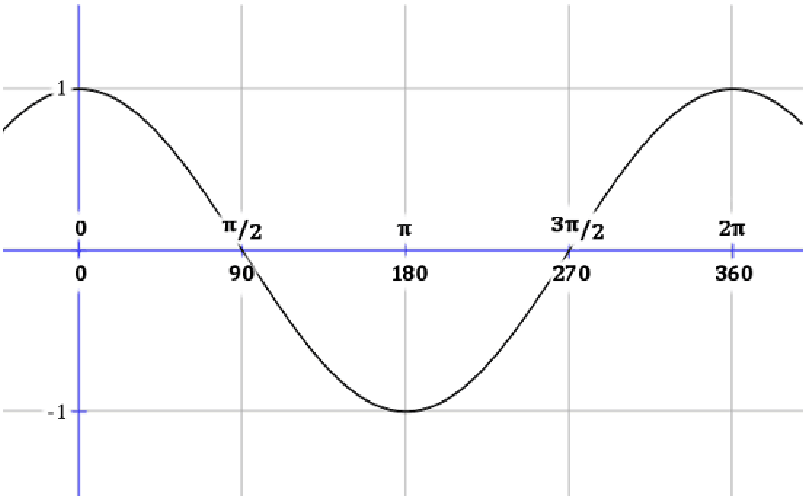
$180° = \pi$ $360° = 2\pi$

4 Graphs

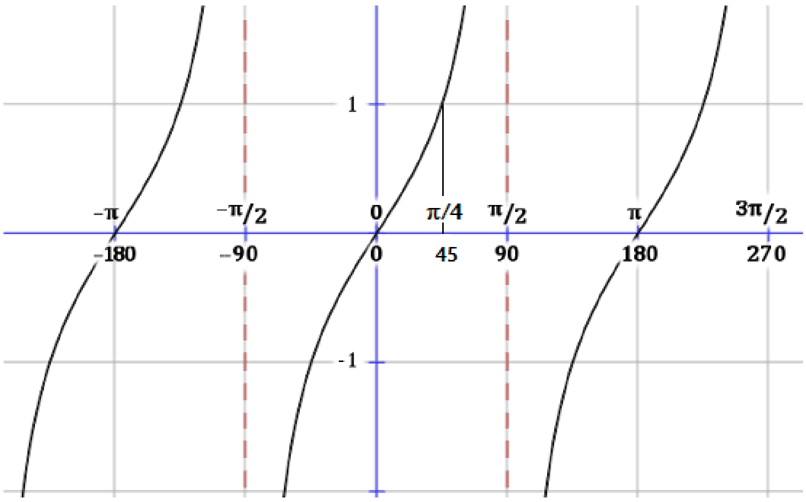
$y = \sin\theta$, no asymptotes



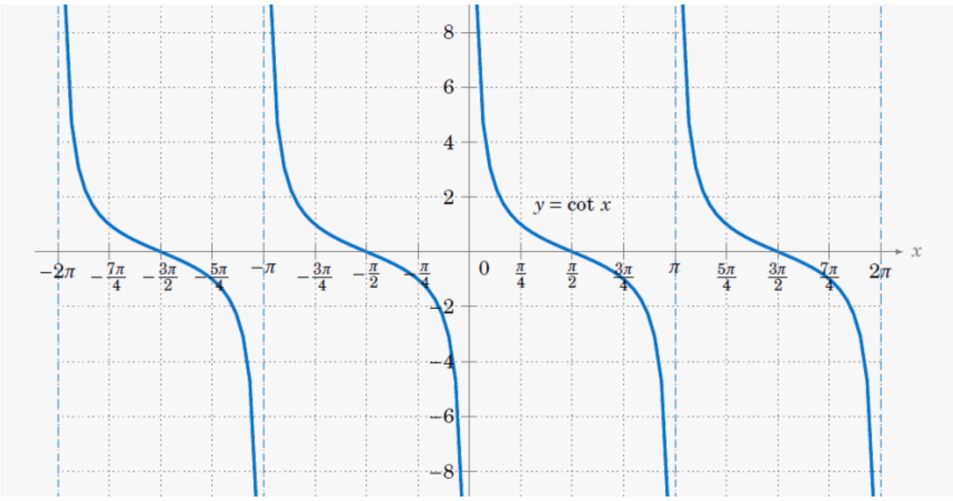
$y = \cos\theta$, no asymptotes



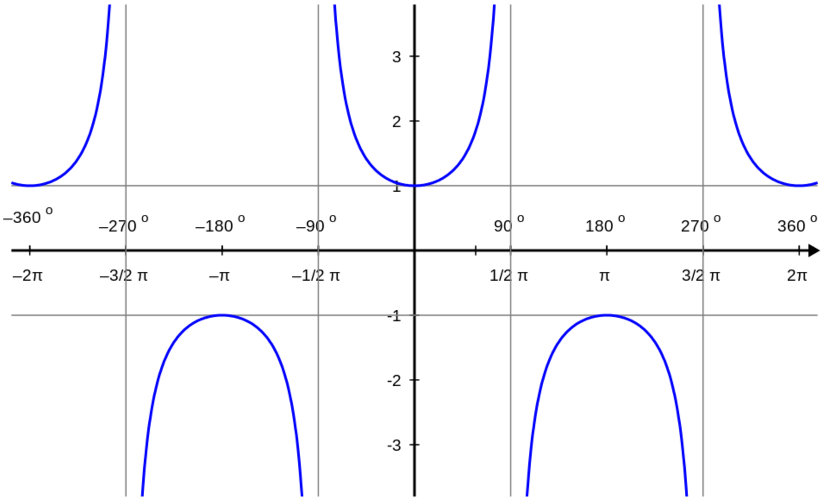
$y = \tan\theta$, asymptotes x = 90°, 270°, etc



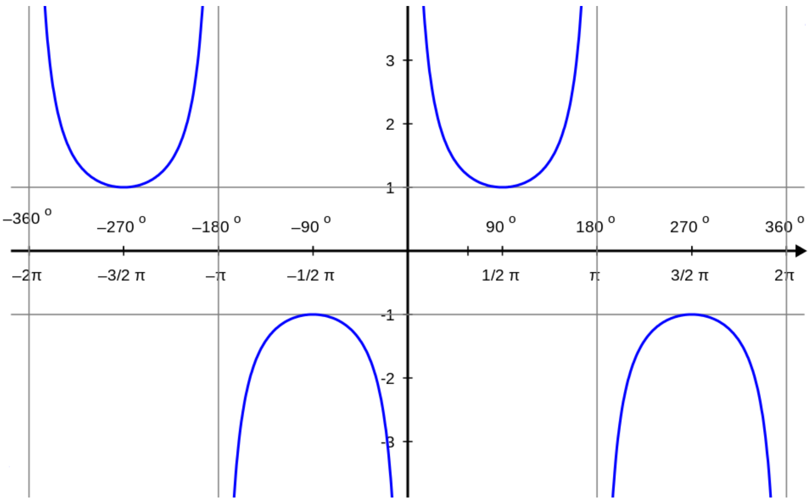
$y = \cot\theta$, asymptotes x = 0°, 180°, 360°, etc



$y = \sec\theta$ asymptotes x = 90°, 270°, etc



$y = \operatorname{cosec}\theta$, asymptotes x = 0°, 180°, 360°, etc



5 Tables

$y = \sin\theta$

X	Y
0°	0
90°	1
180°	0
270°	-1
360°	0

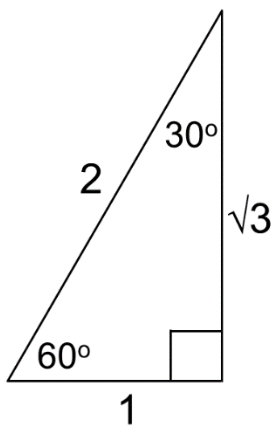
$y = \cos\theta$

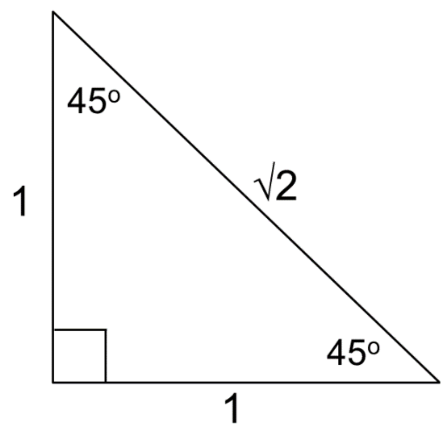
X	Y
0°	1
90°	0
180°	-1
270°	0
360°	1

$y = \tan\theta$

X	Y
0°	0
45°	1
90°	Und
135°	-1
180°	0
225°	1
270°	Und
315°	-1
360°	0

6 Standard Triangles





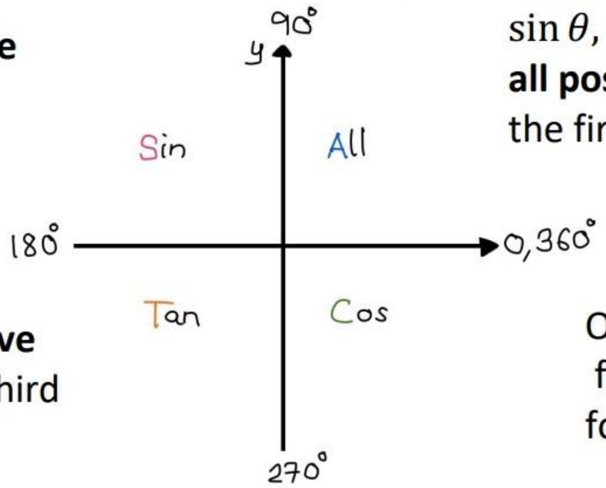
6.1 Exact Values of Trig Ratios

	0°	30°	45°	60°	90°
sin(θ)	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos(θ)	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan(θ)	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Und

7 CAST

Only **sin θ is positive** for angle θ in the second quadrant.

$\sin\theta$, $\cos\theta$ and $\tan\theta$ are **all positive** for angle θ in the first quadrant.



Only **tan θ is positive** for angle θ in the third quadrant

Only **cos θ is positive** for angle θ in the fourth quadrant.

8 Identities

8.1 Supplementary Angles

$sin(\pi - \theta) = sin(\theta)$ $\quad$ $cosec(\pi - \theta) = cosec(\theta)$

$cos(\pi - \theta) = -cos(\theta)$ $\quad$ $sec(\pi - \theta) = -sec(\theta)$

$tan(\pi - \theta) = -tan(\theta)$ $\quad$ $cot(\pi - \theta) = -cot(\theta)$

$sin(\pi + \theta) = -sin(\theta)$ $\quad$ $cosec(\pi + \theta) = -cosec(\theta)$

$cos(\pi + \theta) = -cos(\theta)$ $\quad$ $sec(\pi + \theta) = -sec(\theta)$

$tan(\pi + \theta) = tan(\theta)$ $\quad$ $cot(\pi + \theta) = cot(\theta)$

8.2 Fundamental Identities

$tan\ \theta = \frac{\sin\ \theta}{cos\ \theta}$

$cot\ \theta = \frac{cos\ \theta}{\sin\ \theta}$

$sin^2 \theta\ +\ cos^2 \theta\ \ = 1$

$tan^2 \theta\ + 1 =\ sec^2 \theta$

$cot^2 \theta\ +\ 1 =\ cosec^2 \theta$

$sin^2 \theta\ +\ cos^2 \theta\ \ = 1$

8.3 Compound Angle Identities

$sin(A + B) = sin(A)\, cos(B) + sin(B)\, cos\ (A)$

$sin(A - B) = sin(A)\, cos(B) - sin(B)\, cos\ (A)$

$cos(A + B) = cos(B)\, cos(A) - sin(A)\, sin(B)$

$cos(A - B) = cos(B)\, cos(A) + sin(A)\, sin(B)$

$$tan(A + B) = \frac{\tan(A) + tan(B)}{1 - \tan(A)\tan\ (B)}$$

$$tan(A - B) = \frac{\tan(A) - tan(B)}{1 + \tan(A)\tan\ (B)}$$

8.4 Double Angle Identities

$sin(2A) = 2sin(A)\, cos(A)$

$cos(2A) = cos(A)\, cos(A) - sin(A)\, sin(A)$

$=\ cos^2(A) -\ sin^2(A)$

$=\ 2cos^2(A) -\ 1$

$= 1 -\ 2\, sin^2(A)$

$$tan(2A) = \frac{2\tan(A)}{1 - tan^2(A)}$$

8.5 Product to Sum Identities (use not learn)

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

8.6 Sum to Product Identities (use not learn)

$\sin\alpha + \sin\beta = 2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

$\sin\alpha - \sin\beta = 2\cos(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$

$\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

$\cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$

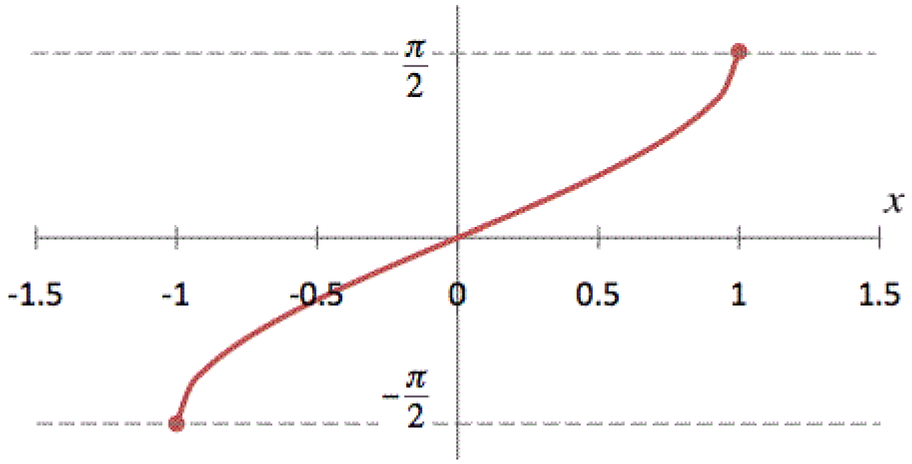
9 Formulae

$Rsin(x \pm \alpha) = asinx \ \pm bcosx$

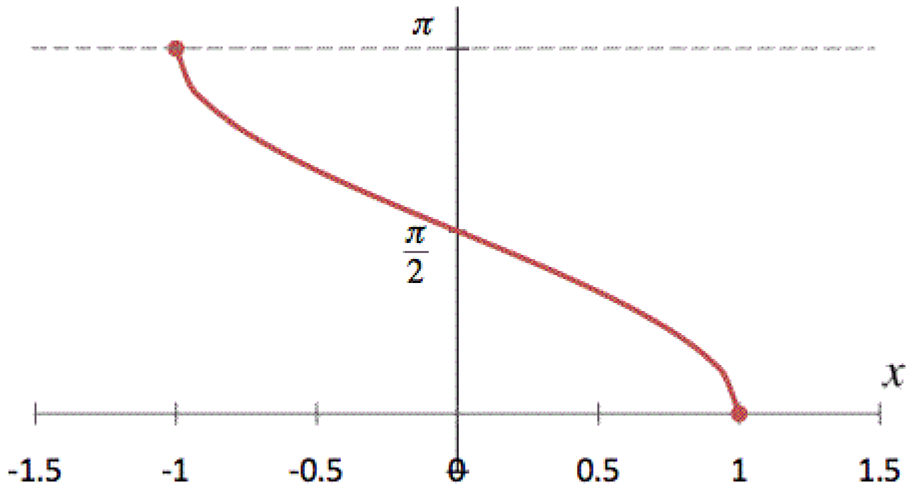
$Rcos(x \mp \alpha) = acosx \ \pm bsinx$

10 Inverses

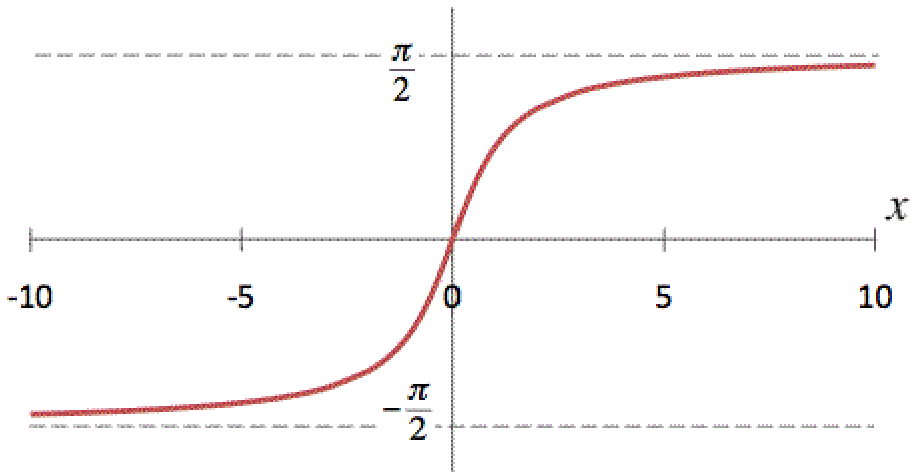
$y = \ sin^{-1}(x) = arcsin(x); \ Domain \ -1 \ \leq x \ \leq 1; \ Range \ -\frac{\pi}{2} \leq y \ \leq \frac{\pi}{2}$



$y = cos^{-1}(\theta) = arccos(\theta);\ Domain\ -1 \le x \le 1;\ Range\ 0 \le y \le \pi$



$y = tan^{-1}(\theta) = arctan(\theta);\ Domain - \infty \le x \le \infty;\ Range\ 0 \le y \le \pi$



11 Rules

11.1 Law of Sines

$$\frac{A}{\sin(A)} = \frac{B}{\sin(B)} = \frac{C}{\sin(C)}$$

$$\frac{\sin(A)}{A} = \frac{\sin(B)}{B} = \frac{\sin(C)}{C}$$

Remember the ambiguous case i.e., two solutions

11.2 Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$